Mixed-variable fuzzy clustering approach to part family and machine cell formation for GT applications

Miin-Shen Yang\textsuperscript{a,}* , Wen-Liang Hung\textsuperscript{b} , Fu-Chou Cheng\textsuperscript{a}

\textsuperscript{a}Department of Applied Mathematics, Chung Yuan Christian University, Chung-Li, Taiwan 32023, ROC
\textsuperscript{b}Department of Mathematics Education, National Hsinchu Teachers College, Hsin-Chu, Taiwan, ROC

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Abstract

Group technology (GT) is a useful way to increase productivity with high quality in flexible manufacturing systems. Cell formation (CF) is a key step in GT. It is used to design a good cellular manufacturing system that uses the similarity measure between parts and machines so that it can identify part families and machine groups. Recently, fuzzy clustering has been applied in GT because the fuzzy clustering algorithm can present partial memberships for part–machine cells so that it is suitably used in cellular manufacturing systems for a variety of real cases. However, these fuzzy clustering algorithms are only used for numeric data of parts and machines. In this paper, we apply a mixed-variable fuzzy clustering algorithm, called mixed-variable fuzzy \(c\)-means (MVFCM), to CF in GT. According to real application examples, the MVFCM algorithm gives good results when it is applied to the mixed-variable types of CF data.

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1. Introduction

High manufacturing profit comes from lower cost and higher product quality. Generally, there are some guidelines for reducing product cost with high quality. To create a good production process is an important step. It needs various kinds of machines and also a lot of complicated work procedures to finish the manufacturing. Frequently, the parts have to move from one place to another. The action may cause not only machine idleness but also consumption of moving time and manpower. On the other hand, there are more and more manufacturing companies that need to carry out batch manufacturing with small or medium production lot size. In this situation, more setup changes and frequent part or machine travel occur. Group technology (GT) has been a useful way to treat these problems with a flexible manufacturing process. It is used to exploit similarities between components to achieve lower cost and to increase productivity with high quality. Cell formation (CF) is a key step in GT. It is used to design a good cellular manufacturing system that uses the similarities of parts related to machines so that it can identify part families and machine groups. The parts in the same machine group have

\*Corresponding author. Tel.: +886 3 456 3171; fax: +886 3 456 3160. 
E-mail address: msyang@math.cycu.edu.tw (M.-S. Yang).
similar requirement so that GT can reduce travel and setup time. In CF, the part/machine matrix which has \( m \times p \) dimensions with binary components is usually described and given. The \( m \) rows indicate \( m \) machines and the \( p \) columns represent \( p \) parts that need to be operated upon. In the matrix, “1” (“0”) represents that this part should be (not) worked on the machine. The matrix exhibits parts requirement relative to machines. Our objective is to group parts and machines just like a cell.

There are many CF methods in the literature (see Singh, 1993; Singh and Rajamani, 1996; Molleman et al., 2002). Some of them used algorithms with certain energy functions or codes to immediately arrange a part/machine matrix, such as the bond energy algorithm (BEA) (McCormick et al., 1972), the rank order clustering (ROC) (King, 1980), the modified rank order clustering (MODROC) (Chandrasekaran and Rajagopalan, 1986), the direct clustering algorithm (DCA) (Chan and Milner, 1982) and evolutionary algorithms (EAs) (Plaquin and Pierreval, 2000). Some others used the similarity-based hierarchical clustering (Mosier, 1989; Wei and Kern, 1989; Gupta and Seifoddini, 1990; Shafer and Rogers, 1993; Liao et al., 1998), for example, single linkage clustering (SLC), complete linkage clustering (CLC), average linkage clustering (ALC), and linear cell clustering (LLC). However, these CF methods assumed well-defined boundaries between part–machine cells. These crisp boundary assumptions may fail to fully describe the case where the part–machine cell boundaries are fuzzy. This is why fuzzy clustering algorithms were applied for CF. Xu and Wang (1989) first considered the fuzzy clustering in the application to CF. Chu and Hayya (1991) then improved its usage. Gindy et al. (1995) considered its optimal cluster number using some validity indexes. Venugopal (1999) gave a state-of-the-art review on the use of soft computing including fuzzy clustering. Moreover, Gungor and Arikan (2000) applied fuzzy decision making in CF. However, these fuzzy clustering methods all considered numeric data. In fact, part or machine data are often presented in symbolic and fuzzy variable types, and these fuzzy clustering algorithms cannot be used for these mixed-variable data.

Symbolic data are different from numeric data. Symbolic variables may present human knowledge, nominal, categorical and synthetic data. Cluster analysis for symbolic data with hierarchical, hard and fuzzy clustering algorithms have been widely studied (see Gowda and Diday, 1992; Gowda and Ravi, 1995). Fuzzy data are another type with imprecise or with a source of uncertainty not caused by randomness, such as linguistic assessments. For example, Liao (2001) proposed classification and coding approaches to part family under a fuzzy environment with the construction of a coding structure for fuzzy data by considering a fuzzy feature as a linguistic variable. The fuzzy data type is easily found in natural language, social science, and knowledge representation. Fuzzy numbers are used to model the fuzziness of data and usually used to represent fuzzy data. There are several papers (see Hathaway et al., 1996; El-Sonbaty and Ismail, 1998) that gave fuzzy clustering algorithms for these fuzzy data.

In real situations, there are data sets with mixed-variable types of symbolic and fuzzy data. Recently, Yang et al. (2004) proposed a fuzzy clustering algorithm for these mixed types of data, called the mixed-variable fuzzy c-means (MVFCM). In this paper, we shall apply this mixed-type data clustering algorithm to CF. The distance measure for symbolic data and fuzzy data defined by Yang et al. (2004) will be used in a part–machine matrix. We then apply the MVFCM algorithm to these mixed-variable types of data. In Section 2, we present fuzzy clustering methods to a binary part–machine matrix with numeric data. In Section 3, we review the definition of symbolic and fuzzy data and give the distance measure for these mixed-variable data. Section 4 gives the MVFCM algorithm for CF. In Section 5, we associate validity indexes to find optimal cell numbers for CF. Section 6 presents examples, and conclusions are stated in Section 7.

## 2. Fuzzy clustering for cell formation

In CF, a part–machine matrix, which has \( n \times m \) dimensions with binary data, is usually given. Assume that there are \( n \) parts and \( m \) machines to be grouped into \( c \) part–machine cells. We can first group \( n \) parts into \( c \) part families and then use part family centers as references to form machine cells. Finally, we reallocate part families and machine cells to form final part–machine cells.

Let \( X = \{x_1, \ldots, x_n\} \) be a set of \( n \) parts where \( x_j = (x_{j1}, \ldots, x_{j\ell}) \) is the \( j \)th part feature vector such that

\[
x_{j\ell} = \begin{cases} 1 & \text{if the part } j \text{ requires the machine } \ell, \\ 0 & \text{otherwise.}
\end{cases}
\]

Thus, \( c \) part families consist of a partition \( X_1, \ldots, X_c \) of \( X \) where \( X_1, \ldots, X_c \) are mutually
disjoint subsets with \( X_1 \cup \cdots \cup X_c = X \). Equivalently, the partition \( \{X_1, \ldots, X_c\} \) can be denoted by the indicator functions \( \mu_1, \ldots, \mu_c \) such that \( \mu_i(x) = 1 \) if \( x \in X_i \) and \( \mu_i(x) = 0 \) if \( x \notin X_i \) for all \( i = 1, \ldots, c \). Usually, \( \{\mu_1, \ldots, \mu_c\} \) is called a hard \( c \)-partition of \( X \). Traditionally, part family formation in GT is to find an optimal hard \( c \)-partition \( \{\mu_1, \ldots, \mu_c\} \) based on a variety of methods, such as coding, product flow analysis, and ROC.

However, this crisp membership of part \( x_j \) belonging to family \( i \) with values 0 or 1 may not be fitted in reality. In general, partial membership with values in the interval \([0,1]\) may be fitted in most real cases. The idea of a membership function with fuzzy set was first used in part family formation by Xu and Wang (1989). They used the well-known fuzzy \( c \)-means (FCM) clustering algorithm to GT application.

Now consider an extension to allow \( \mu_i(x) \) to be a membership function of fuzzy set \( \mu_i \) or \( X \), assuming values in the interval \([0,1]\), such that \( \sum_{i=1}^c \mu_i(x) = 1 \) for all \( x \in X \). In this fuzzy extension, \( \{\mu_1, \ldots, \mu_c\} \) is called a fuzzy \( c \)-partition of \( X \). Thus, the FCM clustering algorithm is based on the minimization of the sum of squared error

\[
J_{\text{FCM}}(\mu, a) = \sum_{i=1}^c \sum_{j=1}^n \mu_{ij}^m \|x_j - a_i\|^2,
\]

where \( m > 1 \) is the index of fuzziness, \( a_i \) is the cluster center of the part family \( k \), and \( \mu_{ij} = \mu_i(x_j) \). The necessary conditions for a minimizer \((\mu, a)\) of \( J_{\text{FCM}} \) are the following update equations:

\[
\mu_{ij} = \left( \frac{\sum_{k=1}^c \|x_j - a_k\|^{2/(m-1)}}{\sum_{k=1}^c \|x_j - a_k\|^{2/(m-1)}} \right)^{-1},
\]

and

\[
a_i = \frac{\sum_{j=1}^n \mu_{ij}^m x_j}{\sum_{j=1}^n \mu_{ij}^m}.
\]

The FCM clustering algorithm and its variety of generalizations have been widely studied and applied (see Bezdek, 1981; Yang, 1993, Wu and Yang (2002)). This FCM algorithm was first used in part-family formation by Xu and Wang (1989). Chu and Hayya (1991) formed part families based on FCM membership functions \( \mu_1, \ldots, \mu_c \) and then extensively considered the machine cell configuration on the basis of FCM part family centers. Finally, they reallocated the final classification part–machine matrix to final part–machine cells.

They demonstrated that the method was sensitive to the family number \( c \), the index of fuzziness \( m \), and also to the stopping criterion of the algorithm. The family number \( c \) in FCM is supposed to be known a priori. However, we do not have a priori knowledge of \( c \) in most real cases. To solve this problem, Gindy et al. (1995) proposed a validity measure for part family formation in GT applications. To improve performance, Josien and Liao (2000) proposed an integrated use of FCM and fuzzy \( k \)-nearest neighbor (KNN) rule. The results from FCM are then used as an input to the supervised fuzzy KNN as training data. The approach improves the performance of part–machine CF.

The FCM clustering algorithm had been successfully used in part–machine CF. However, FCM can be only used for numeric type part–machine data. In real cases, part–machine data may appear in different types, such as numeric, symbolic, or fuzzy. On the other hand, the part–machine matrix may have multiple attributes for parts on machine. In some situations, part data may include several different attributes, and similarly for machines. To solve these mixed-variable types in part–machine CF, we use an extended type of FCM, called mixed-variable FCM (MVFCM) (see Yang et al., 2004).

3. Mixed-variable data and distances

The numeric (or vector) data are used for most cases. However, except for these numeric data, there are many other types of data such as symbolic and fuzzy. In this section, we consider the mixed feature type of the numeric, symbolic, and fuzzy data. The distance for these mixed feature data defined by Yang et al. (2004) will be adopted here.

Suppose that a feature vector \( F \) is written as a \( d \)-tuple of feature components \( F_1, \ldots, F_d \), with

\[
F = F_1 \times \cdots \times F_d.
\]

For any two feature vectors \( A \) and \( B \) with \( A = A_1 \times \cdots \times A_d \) and \( B = B_1 \times \cdots \times B_d \), the distance between \( A \) and \( B \) is defined as

\[
d(A, B) = \sum_{k=1}^d d(A_k, B_k),
\]

where \( d(A_k, B_k) \) represents the dissimilarity of the \( k \)th feature component according to its feature type. Because there are symbolic and fuzzy feature components in a \( d \)-tuple feature vector, the distance
$d(A, B)$ will be the sum of the dissimilarity of symbolic data, which is defined by $d_p(A_k, B_k)$ for the “position”, $d_s(A_k, B_k)$ for the “span” and $d_c(A_k, B_k)$ for the “content”, and also with the dissimilarity of fuzzy data which is defined by $d_f(A_k, B_k)$. Thus, $d(A, B)$ can include $d_p(A_k, B_k), d_s(A_k, B_k), d_c(A_k, B_k)$ and $d_f(A_k, B_k)$.

3.1. Symbolic feature components

Various definitions and descriptions of symbolic objects were given by Diday (1988). According to Gowda and Diday (1992), symbolic features can be divided into quantitative and qualitative features in which each feature can be defined by $d_p(A_k, B_k)$ due to position $p$, $d_s(A_k, B_k)$ due to span $s$ and $d_c(A_k, B_k)$ due to content $c$. They then defined a distance for these symbolic data. Recently, Yang et al. (2004) gave an improved distance measure by modifying the Gowda and Diday’s dissimilarity definition as follows.

3.1.1. Quantitative type of $A_k$ and $B_k$

The distance between two feature components of quantitative type is defined as the dissimilarity of these values due to position, span, and content.

Let $a_l = \text{lower limit of } A_k$;
$a_u = \text{upper limit of } A_k$;
$b_l = \text{lower limit of } B_k$;
$b_u = \text{upper limit of } B_k$;
inters = length of intersection of $A_k$ and $B_k$;
$s_l = \text{span length of } A_k$ and
$s_b = |\max(a_u, b_u) - \min(a_l, b_l)|$;
$U_k = \text{the difference between the highest and lowest values of the } 4\text{th feature over all objects};$

$$l_a = |a_u - a_l|;$$
$$l_b = |b_u - b_l|.$$  

The three distance components are then defined as follows:

due to position: $d_p(A_k, B_k) = \frac{|a_u+a_l-b_u-b_l|}{U_k}$,

due to span: $d_s(A_k, B_k) = \frac{|l_a - l_b|}{U_k + l_a + l_b - \text{inters}}$,

due to content: $d_c(A_k, B_k) = \frac{|l_a + l_b - 2 \cdot \text{inters}|}{U_k + l_a + l_b - \text{inters}}$.

Thus, the distance $d(A_k, B_k)$ is defined as

$$d(A_k, B_k) = d_p(A_k, B_k) + d_s(A_k, B_k) + d_c(A_k, B_k).$$

3.1.2. Qualitative type of $A_k$ and $B_k$

For qualitative feature types, the dissimilarity component due to position is absent. The term $U_k$ for qualitative feature types is absent too. The two components that contribute to dissimilarity are “due to span” and “due to content”.

Let $l_a = \text{length of } A_k = \text{the number of elements in } A_k$;
$l_b = \text{length of } B_k = \text{the number of elements in } B_k$;
$s_l = \text{length of } A_k$ and $B_k = \text{the number of elements in the union of } A_k$ and $B_k$;
inters = the number of elements in the intersection of $A_k$ and $B_k$.

The two dissimilarity components are then defined as follows:

$$d_s(A_k, B_k) = \frac{|l_a - l_b|}{l_s},$$

$$d_c(A_k, B_k) = \frac{|l_a + l_b - 2 \cdot \inters|}{l_s}.$$  

Thus, the distance $d(A_k, B_k)$ for qualitative type $A_k$ and $B_k$ is defined as

$$d(A_k, B_k) = d_s(A_k, B_k) + d_c(A_k, B_k).$$

3.2. Fuzzy feature components

Fuzzy data types often appear in real applications. Fuzzy numbers are used to model the fuzziness of data and are usually used to represent fuzzy data. Trapezoidal fuzzy numbers (TFN) are used most. Hathaway et al. (1996) proposed the FCM clustering algorithm for symmetric TFN, using a parametric approach. They defined a dissimilarity for two symmetric TFNs and then used it for FCM clustering. However, they did not consider the left or right shapes of fuzzy numbers (i.e. LR-type TFN). Yang et al. (2004) gave a distance for LR-type TFNs based on Yang and Ko’s distance definition (1996) as follows.

Hathaway et al.’s parametric approach gave the parameterization of a TFN $A$ as $A = (a_1, a_2, a_3, a_4)$, where we refer to $a_1$ as the center, $a_2$ as the inner diameter, $a_3$ as the left outer radius, and $a_4$ as the right outer radius. Using this parametric representation we can parameterize the four kinds of TFNs with real numbers, intervals, triangular, and TFN. Let $L$ (and $R$) be decreasing, shape functions from $R^+$ to $[0, 1]$, with $L(0) = 1$; $L(x) < 1$, for all $x > 0$; $L(x) > 0$, for all $x < 1$; $L(1) = 0$ or $L(x) > 0$, for all $x,$
Given \( A \) and \( B \), Hathaway’s parametric representation, 
where \( \alpha > 0 \) and \( \beta > 0 \) are called the left and right spreads, respectively. 
Given \( A = ((m_1 + m_2)/2, m_2 - m_1, \alpha, \beta) \) and \( B = ((m_2 + m_2)/2, m_2 - m_2, \alpha, \beta) \), Hathaway et al. (1996) defined the distance \( d_{LR}(A, B) \) of \( A \) and \( B \) as

\[
d_{LR}(A, B) = \frac{(a_1 - a_2)^2 + (b_1 - b_2)^2}{2} + \frac{(a_2 - a_1)^2 + (b_2 - b_1)^2}{2} + \frac{(a_1 - a_1)^2 + (b_1 - b_1)^2}{2} + \frac{(a_2 - a_2)^2 + (b_2 - b_2)^2}{2},
\]

where \( l = \int_0^1 L^{-1}(w)dw \) and \( r = \int_0^1 R^{-1}(w)dw \). If \( L \) and \( R \) are linear, then \( l = r = \frac{1}{2} \). Thus, for any given two TFNs \( A = (m_1, a_1, a_2, a_3, a_4) \) and \( B = (m_2, b_1, b_2, b_3, b_4) \), we have a distance \( d_{LR}(A, B) \) on the basis of Yang et al.’s definition with

\[
d_{LR}(A, B) = \frac{(a_1 - a_2)^2 + (b_1 - b_2)^2}{2} + \frac{(a_1 + a_2)^2 + (b_1 + b_2)^2}{2} + \frac{(a_2 - a_1)^2 + (b_2 - b_1)^2}{2} + \frac{(a_1 - a_2)^2 + (b_1 - b_1)^2}{2} + \frac{(a_2 - a_2)^2 + (b_2 - b_2)^2}{2},
\]

Then

\[
d_{LR}(A, B) = \frac{1}{4} (g_+^2 + g_-^2 + (g_- - (a_3 - b_3))^2 + (g_+ - (a_4 - b_4))^2),
\]

where \( g_- = 2(a_1 - b_1) - (a_2 - b_2) \) and \( g_+ = 2(a_1 - b_1) + (a_2 - b_2) \).

The distance \( d_l \) is different from Hathaway et al.’s (1996) distance. The main difference is that the distance \( d_l(A, B) \) takes the shape functions \( L \) and \( R \) into account, but Hathaway et al.’s does not, which means that the distance \( d_l(A, B) \) includes information of shape functions \( L \) and \( R \) in the distance measures. Yang et al. (2004) have illustrated these differences.

4. Mixed-variable fuzzy clustering algorithm for cell formation

Let \( X = \{x_1, \ldots, x_n\} \) be a set of \( n \) part feature vectors. If part feature vectors are in \( \mathbb{R}^q \), then the FCM algorithm can be used to group part families based on the update Eqs. (1) and (2). However, in applying FCM to symbolic or fuzzy data, there are problems encountered; for example, the weighted mean equation and the Euclidean distance \( \| \cdot \| \) are not suitable for these mixed-feature data. To overcome these problems, a new representation for cluster centers should be used (e.g., El-Sonbaty and Ismail, 1998; Yang et al., 2004).

A cluster center is assumed to be formed as a group of features, and each feature is composed of several events. Let \( A_{kp|i} \) be the \( p \)th event of feature \( k \) in cluster \( i \) and let \( e_{kp|i} \) be the membership degree of association of the \( p \)th event \( A_{kp|i} \) to the feature \( k \) in cluster \( i \). Thus, the \( k \)th feature of the \( i \)th cluster center \( A_i \) can be presented as

\[
A_k = [(A_{k1|i}, e_{k1|i}), \ldots, (A_{kp|i}, e_{kp|i})].
\]

In this case, we shall have

\[
0 \leq e_{kp|i} \leq 1 \quad \text{and} \quad \sum_p e_{kp|i} = 1,
\]

\[
\bigcap_p A_{kp|i} = \emptyset \quad \text{and} \quad \bigcup_p A_{kp|i} = \bigcup_j X_j,
\]

where \( e_{kp|i} = 0 \) if the event \( A_{kp|i} \) is not a part of feature \( k \) in the cluster center \( A_i \) and \( e_{kp|i} = 1 \) if there are no other events than the event \( A_{kp|i} \) sharing this event in forming the feature \( k \) in cluster center \( A_i \). Thus, the update equation for \( e_{kp|i} \) is

\[
e_{kp|i} = \frac{\sum_{j=1}^n \mu_{ij}^m \theta}{\sum_{j=1}^n \mu_{ij}^m}, \tag{4}
\]

where \( \theta \in \{0, 1\} \) and \( \theta = 1 \) if the \( k \)th feature of the \( j \)th datum \( X_j \) consists of the \( p \)th event; otherwise \( \theta = 0 \). \( \mu_{ij} = \mu_i(X_j) \) is the membership of \( X_j \) in cluster \( i \).
The membership function $e_{kp|i}$ is an important index function for using FCM with symbolic data. Except for applying the FCM to symbolic feature components, we also consider the FCM in fuzzy feature components. Let $\{X_1, \ldots, X_n\}$ be a mixed-feature data set. Then, the FCM objective function can be defined as

$$J(\mu, e, a) = \sum_{i=1}^{c} \sum_{j=1}^{n} \mu_{ij}^{m} d^2(X_j, A_i),$$

where

$$d^2(X_j, A_i) = \sum_{k' \text{ of symbolic}} \left( \sum_{\rho} d^2(X_{jk'}, A_{k'|i}) e_{k'|i} \right) + \sum_{k \text{ of fuzzy}} d^2_j(X_{jk}, A_{ik}).$$

There are parameters $\{\mu_1, \ldots, \mu_c\}, \{e_{k'|i}\}, \{a_{ik1}, a_{ik2}, a_{ik3}, a_{ik4}\}$ for the FCM objective function $J(\mu, e, a)$. The Picard method (see Carothers et al. (2004)) for approximating the optimal solutions which minimize the objective function $J(\mu, e, a)$ is considered with the necessary condition of a minimizer of $J$ over the parameter $\mu$. The update equation for the membership functions is

$$\mu_{ij} = \left( \sum_{q=1}^{c} \left( \frac{d^2(X_j, A_i)}{d^2(X_{jk'}, A_{k'|i})} \right)^{1/(m-1)} \right)^{-1},$$

$$i = 1, \ldots, c, \quad j = 1, \ldots, n,$$

where $d^2(X_j, A_i)$ is defined by Eq. (6) for which $d^2(X_{jk'}, A_{k'|i})$ and $d^2_j(X_{jk}, A_{ik})$ are the dissimilarities for symbolic and fuzzy data proposed in Section 2. According to Yang et al. (2004), there are two groups of parameters $e$ and $a$ to be considered where the parameters $e_{k'|i}$ are for these $k'$ that are symbolic and the parameters $\{a_{ik1}, a_{ik2}, a_{ik3}, a_{ik4}\}$ are for these $k$ that are fuzzy. Thus,

(a) For these $k'$ that are symbolic, we have the following update equation:

$$e_{k'|i} = \frac{\sum_{j=1}^{n} \mu_{ij}^{m}}{\sum_{j=1}^{n} \mu_{ij}^{m}}.$$  

(b) For these $k$ that are fuzzy, we have the following update equations:

$$a_{ik1} = \frac{\sum_{j=1}^{n} \mu_{ij}^{m} (8x_{jk1} - x_{jk3} + x_{jk4} + a_{ik3} - a_{ik4})}{8\sum_{j=1}^{n} \mu_{ij}^{m}},$$

$$a_{ik2} = \frac{\sum_{j=1}^{n} \mu_{ij}^{m} (4x_{jk2} + x_{jk3} + x_{jk4} - a_{ik3} - a_{ik4})}{4\sum_{j=1}^{n} \mu_{ij}^{m}},$$

$$a_{ik3} = \frac{\sum_{j=1}^{n} \mu_{ij}^{m} (-2x_{jk1} + x_{jk2} + x_{jk3} + 2a_{ik1} - a_{ik2})}{\sum_{j=1}^{n} \mu_{ij}^{m}},$$

$$a_{ik4} = \frac{\sum_{j=1}^{n} \mu_{ij}^{m} (2x_{jk1} + x_{jk2} + x_{jk4} - 2a_{ik1} - a_{ik2})}{\sum_{j=1}^{n} \mu_{ij}^{m}}.$$  

On the basis of these necessary conditions we can construct an iterative algorithm.

**Mixed-variable FCM (MVFCM):**

**Step 1:** Fix $m$ and $c$ and give an $e > 0$. Initialize a fuzzy $c$-partition $\mu^{(0)} = \{\mu_{1}^{(0)}, \ldots, \mu_{c}^{(0)}\}$ and set $\ell = 0$.

**Step 2:** For symbolic feature $k'$, compute the $i$th cluster center $A_{ik'}^{(\ell)} = [A_{k'|i}^{(\ell)}, e_{k'|i}^{(\ell)}], \ldots, [A_{ik2}^{(\ell)}, e_{k2}^{(\ell)}]$, using Eq. (8). For fuzzy feature $k$, compute the $i$th cluster center $A_{ik}^{(\ell)} = \{a_{ik1}, a_{ik2}, a_{ik3}, a_{ik4}\}$, using Eqs. (9)–(12).

**Step 3:** Update $\mu^{(\ell+1)}$ using Eq. (7).

**Step 4:** Compare $\mu^{(\ell+1)}$ to $\mu^{(\ell)}$ in a convenient matrix norm.

- **IF** $\|\mu^{(\ell+1)} - \mu^{(\ell)}\| < \epsilon$, **THEN** STOP.
- **ELSE** $\ell = \ell + 1$ and GOTO Step 2.

In GT, one may have part families based on part feature vectors. Another is based on the consideration of production flow analysis (PFA) (Burbidge, 1991). In PFA, a part–machine matrix is assumed where the matrix $X = [x_{jt}]_{n \times m}$ is a binary matrix with $x_{jt} = 1$, if part $j$ requires machine $t$, and $x_{jt} = 0$, otherwise. There are many grouping methods used for these GT applications. According to the review in Venugopal (1999), the approaches to the part–machine grouping problems can be classified as soft computing and traditional hard computing on the basis of the computing viewpoint. However, we find that all of these part–machine grouping approaches are considered for numeric data. In Example 1, we will apply the MVFCM algorithm to part family, when part feature vectors are mixed-variable types of symbolic and fuzzy data.
Example 1. There are 10 parts of forging steels. Each part is described using 6 attributes of A1–A6. The attributes include numeric, symbolic, and fuzzy data. The data are shown in Table 1. The attribute A1 is a categorical type of symbolic data. The attributes A2 and A4 are qualitative types of symbolic data. The attributes A3 and A5 are fuzzy data and the attribute A6 is numeric.

According to a priori knowledge, the cluster number is \( c = 3 \). Because the data in Table 1 are mixed-variable types, we use the MVFCM algorithm to group part families. The results from MVFCM on the data set of Table 1 are shown in Table 2. We find that the part families are \( f_{1,2,3} \); \( f_{4,5,6,7} \); \( f_{8,9,10} \). It gets good results. Although we have a priori knowledge of \( c = 3 \), that is usually unknown in most real cases. Finding an optimal cluster number \( c \) is called a validity problem. Usually, a validity index is used for finding an optimal \( c \). We shall treat this kind of problem in the MVFCM algorithm in the next section so that it can be practically used in GT applications.

5. Optimal cell number in part–machine cell formation

After a fuzzy \( c \)-partition \( \{\mu_1, \ldots, \mu_c\} \) is provided by a fuzzy clustering algorithm, such as FCM or MVFCM, we may ask whether the \( c \)-partition accurately represents the structure of the data set. This is a cluster-validity problem (see Bezdek, 1974b; Yang, 1993). Since most fuzzy clustering methods need to pre-assume the cluster number \( c \), a validity index for finding an optimal \( c \) that can completely describe the data structure becomes a most studied topic in cluster validity. We briefly review the three most used FCM-based indexes and then extend these to the proposed MVFCM.

(a) The first validity index associated with FCM was the partition coefficient (PC) (Bezdek, 1974a) defined by

\[
PC(c) = \frac{1}{n} \sum_{i=1}^{c} \sum_{j=1}^{n} \mu_{ij}^2, \quad (13)
\]

where \( 1/c \leq PC(c) \leq 1 \). An optimal cluster number \( c^* \) is obtained by solving \( \max_{2 \leq c \leq n-1} PC(c) \) to produce the best clustering performance.

(b) The partition entropy (PE) (Bezdek, 1974b) was defined by

\[
PE(c) = -\frac{1}{n} \sum_{i=1}^{c} \sum_{j=1}^{n} \mu_{ij} \log_2 \mu_{ij}, \quad (14)
\]
where $0 \leq \text{PE}(c) \leq \log_2 c$. An optimal $c^*$ is obtained by solving $\min_{2 \leq c \leq n-1} \text{PE}(c)$ to produce a best clustering performance.

(c) A validity function proposed by Xie and Beni (XB) (1991) was defined by

$$\text{XB}(c) = \frac{\sum_{i=1}^{c} \sum_{j=1}^{n} \mu_{ij}^m \sum_{k} \text{symbolic}(\sum_{p} d^2(X_{jk}, A_{k'} p_i) e_{k'} p_i))}{n \min_{i,j} \sum_{k} \text{symbolic}(\sum_{p} d^2(A_{k'} p_i) e_{k'} p_i))}$$

(15)

where $J_m(\mu, a)$ is a compactness measure and $\text{Sep}(a)$ is a separation measure. An optimal $c^*$ is found by solving $\min_{2 \leq c \leq n-1} \text{XB}(c)$ to produce the best clustering performance.

The above three indexes are the most-used validity indexes. The indexes PC and PE can be directly used for MVFCM with mixed-feature data. However, the XB index uses the compactness and separation measures that involve data types, so that it needs to be modified when it is used in MVFCM. Obviously, it can be modified as follows:

$$\text{XB}(c) = \frac{\sum_{i=1}^{c} \sum_{j=1}^{n} \mu_{ij}^m \sum_{k} \text{fuzzy} \left(\sum_{p} d^2(X_{jk}, A_{k'} j) e_{k'} j\right)}{n \min_{i,j} \sum_{k} \text{fuzzy} \left(\sum_{p} d^2(A_{k'} j) e_{k'} j\right)} + \frac{\sum_{i=1}^{c} \sum_{j=1}^{n} \mu_{ij}^m \sum_{k} \text{symbolic} \left(\sum_{p} d^2(X_{jk}, A_{k'}) e_{k'} x\right)}{n \min_{i,j} \sum_{k} \text{symbolic} \left(\sum_{p} d^2(A_{k'} x) e_{k'} x\right)}$$

Next, we continue Example 1 of Section 4 using the four validity indexes.

**Example 1 (Continued).** We run the data set of Table 1 from Example 1 in Section 4 using the indexes PC, PE, and XB. We run these from $c = 2$ to $c = 9$. The results are shown in Table 3. We find that PC and PE give the optimal cluster number $c^* = 2$, and XB gives the optimal cluster number $c^* = 7$. In this case, we choose $c^* = 2$ as its optimal cluster number estimate. We then implement MVFCM for the data set with $c = 2$. The results are shown in Table 4. We have the optimal part families with \{3,4,5,7\} and \{1,2,6,8,9,10\}.

## 6. Applications to part–machine cell formation

We had created an MVFCM algorithm for mixed-variable data in Section 4 and gave an estimate to an optimal cluster number, $c$, based on the validity indexes in Section 5. Example 1 was given to demonstrate its efficiency. In this section, we shall apply MVFCM to part family and machine CF in real cases.

### 6.1. A part family application

We consider a real example about the production of forging steels used in shipbuilding. There are five parts for these forging steels. These are screwshaft, steel bars, square steel, countershaft, and hoist plate. Each part has seven principle attributes: section equivalent, yield point, hardness, tensile strength, and reduction of area. The data set of five parts with mixed-variable data for forging steel used in shipbuilding are shown in Table 5. The first attribute in the mixed-variable data is the section equivalent that describes the size of parts with the measure unit in mm. This section size is the main characteristic of outer geometric description to the parts for forging steels. It is numeric data. The second attribute is the

<table>
<thead>
<tr>
<th>Table 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Indexes results from MVFCM for data in Table 1</strong></td>
</tr>
<tr>
<td><strong>Cluster numbers</strong></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>c = 2</td>
</tr>
<tr>
<td>c = 3</td>
</tr>
<tr>
<td>c = 4</td>
</tr>
<tr>
<td>c = 5</td>
</tr>
<tr>
<td>c = 6</td>
</tr>
<tr>
<td>c = 7</td>
</tr>
<tr>
<td>c = 8</td>
</tr>
<tr>
<td>c = 9</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 4</th>
</tr>
</thead>
<tbody>
<tr>
<td><em><em>Clustering results from MVFCM when c^</em> = 2</em>*</td>
</tr>
<tr>
<td><strong>Parts</strong></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td>7</td>
</tr>
<tr>
<td>8</td>
</tr>
<tr>
<td>9</td>
</tr>
<tr>
<td>10</td>
</tr>
</tbody>
</table>
heat treatment technicality. Different tempered heat treatments actually influence the inner structure of forging steels. It is qualitative-type symbolic data. The third attribute is the carbon equivalent percentage which is an impact factor for the hardness and other components of forging steels. It is quantitative-type symbolic data. The fourth attribute is the yield point. It represents the change point when the material begins to have plastic change. The measure unit is N/mm², denoted by MPa. It is quantitative-type symbolic data. The fifth attribute is hardness. The measure unit is Brinell hardness (HB). It is fuzzy data. The final two attributes are tensile strength and area reduction rate. These are quantitative-type symbolic data. We now use the dissimilarity measure defined in Section 3 and the MVFCM algorithm described in Section 4 to the data set of Table 5. The most complicated calculation is the symbolic data cluster center and its membership. According to Table 5, we know that heat treatment technicality, carbon equivalent score, yield point, tensile strength, and area reduction rate are symbolic data. Next, we demonstrate the calculation about the cluster center.

Assume that the cluster number \( c = 3 \). We give initial membership of five parts to the first cluster as \( \mu_{11} = 0.26, \mu_{12} = 0.02, \mu_{13} = 0.94, \mu_{14} = 0.37, \) and \( \mu_{15} = 0.16 \). We find out the structure of a cluster center for the component of heat treatment technicality. For the cluster center, we have its total with 
\[
0.26 + 0.02 + 0.94 + 0.37 + 0.16 = 1.75.
\]
Thus for the heat treatment technicality, we have the membership of the cluster center with 
830°C normalized, 600°C tempered: 
\[
(0.26 + 0.37)/1.75 = 0.36,
\]
890°C normalized, 600°C tempered: 
\[
(0.02 + 0.16)/1.75 = 0.1029,
\]
860°C normalized, 600°C tempered: 
\[
0.94/1.75 = 0.5371.
\]
Similarly, we can find memberships of other symbolic data components for the cluster centers as shown in Table 6.

When we implement MVFCM, we need to assign the cluster number \( c \). Based on the discussion in Section 5, we can use the validity indexes to find an optimal \( c \). Thus, we implement MVFCM for the data of Table 5 and then find the validity indexes for

<table>
<thead>
<tr>
<th>Forging parts</th>
<th>Section equivalent</th>
<th>Heat treatment technicality</th>
<th>Carbon equivalent</th>
<th>Yield point</th>
<th>Hardness</th>
<th>Tensile strength</th>
<th>Area reduction rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Screwshaft</td>
<td>780</td>
<td>830°C C normalized, 600°C tempered</td>
<td>0.48–0.56</td>
<td>340</td>
<td>[1,30,15,0]</td>
<td>655</td>
<td>40</td>
</tr>
<tr>
<td>Steel bar</td>
<td>400</td>
<td>890°C C normalized, 600°C tempered</td>
<td>0.16–0.23</td>
<td>265</td>
<td>[1,10,25,0]</td>
<td>455</td>
<td>50</td>
</tr>
<tr>
<td>Square steel</td>
<td>500</td>
<td>860°C C normalized, 600°C tempered</td>
<td>0.37–0.45</td>
<td>265</td>
<td>[1,25,0,10]</td>
<td>545</td>
<td>45</td>
</tr>
<tr>
<td>Idler or countershaft</td>
<td>680</td>
<td>830°C C normalized, 600°C tempered</td>
<td>0.48–0.56</td>
<td>350</td>
<td>[1,30,15,0]</td>
<td>690</td>
<td>40</td>
</tr>
<tr>
<td>Hoist plate</td>
<td>300</td>
<td>890°C C normalized, 600°C tempered</td>
<td>0.16–0.23</td>
<td>280</td>
<td>[1,10,25,0]</td>
<td>490</td>
<td>50</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Heat treatment technicality</th>
<th>Carbon equivalent</th>
<th>Yield point</th>
<th>Hardness</th>
<th>Tensile strength</th>
<th>Area reduction rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>830°C C normalized, 600°C tempered</td>
<td>0.36</td>
<td>0.1486</td>
<td>0.0114</td>
<td>0.0914</td>
<td>40</td>
</tr>
<tr>
<td>890°C C normalized, 600°C tempered</td>
<td>0.1029</td>
<td>0.5486</td>
<td>0.2114</td>
<td>0.0914</td>
<td>50</td>
</tr>
<tr>
<td>860°C C normalized, 600°C tempered</td>
<td>0.5371</td>
<td>0.1486</td>
<td>0.2114</td>
<td>0.0914</td>
<td>45</td>
</tr>
</tbody>
</table>

Table 5
Five parts with mixed-variable attributes for forging steels used in shipbuilding

Table 6
Structure of symbolic data components for a cluster center

Heat treatment technicality
830°C C normalized, 600°C tempered: 0.36
890°C C normalized, 600°C tempered: 0.1029
860°C C normalized, 600°C tempered: 0.5371

Carbon equivalent score
0.48–0.56: 0.36
0.16–0.23: 0.1029
0.37–0.45: 0.5371

Yield point \( \sigma_s \) (MPa)
340: 0.1486
265: 0.5486
350: 0.2114
280: 0.0914

Tensile strength \( \sigma_b \) (MPa)
655: 0.1486
455: 0.1114
545: 0.5371
690: 0.2114
490: 0.0914

Area reduction rate \( \phi\% \)
40: 40
50: 50
45: 45
$c = 2, 3$ and $4$. The results are shown in Table 7. We find that PC and PE give an optimal $c^* = 2$ and XB gives an optimal $c^* = 3$. In this case, we choose $c = 2$ as its cluster number estimate. For a given $c = 2$, we have the final clustering results shown in Table 8. That is, the parts of screwshaft and countershaft are in the same cluster; and steel bar, square steel and hoist plate are in another cluster. We see that the grouping results are quite reasonable.

### 6.2. A part–machine cell formation application

We consider a real application of the MVFCM algorithm to the production of cast aluminum alloys and forging steels based on eight machine processes where the part–machine matrix has mixed-variable data types. There are seven parts with ZL207, 50SiMn, 35SiMn, ZL205A, ZL402, 42SiMn, and ZL202, and eight machines with eight different processes of fusion, air impermeability, tensile test, cast, impact test, crack-arresting fracture, heat treatment, and cooling. The part–machine matrix is shown in Table 9. We begin to construct part families, machine groups, and part–machine CF for the mixed-variable data shown in Table 9.

We see that each part of these seven cast aluminum alloys and forging steels has eight (machine) attribute components. These attributes are mixed-variable data types so that the MVFCM algorithm can be used for part families. We first implement MVFCM with the validity indexes PC, PE and XB, for $c = 2$ to $c = 6$. The results are shown in Table 10. We find that PE and XB give an optimal $c^* = 4$ and PC gives an optimal $c^* = 5$. In this case, we choose $c = 4$ as its cluster number estimate. We then give the final MVFCM memberships of seven parts for four families $c_1, c_2, c_3,$ and $c_4$. The results are shown in Table 11 with four part families of \{50SiMn, 35SiMn, 42SiMn\}, \{ZL402, ZL202\}, \{ZL207\}, and \{ZL205A\}. We continue working for machine grouping. We see that there are eight machines with seven attribute components from Table 9. However, we find that there are different attribute data types corresponding to the same component of different machines. Say, for example, that the machine 4 has the attribute “S” and that the machine 6 has the attribute “[20,50]” corresponding to the component of ZL402. In those cases, we cannot directly use MVFCM for these machine data. Fortunately, we can transform these mixed-variable machine data to

### Table 7

| Cluster numbers | Validity indexes | | | |
|-----------------|------------------|-----------------|------------------|
|                 | PC($c$)          | PE($c$)         | XB($c$)          |
| $c = 2$         | 0.9356           | 0.1869          | 1.6757           |
| $c = 3$         | 0.8618           | 0.4159          | 0.3787           |
| $c = 4$         | 0.2750           | 1.9624          | 0.4369           |

### Table 8

<table>
<thead>
<tr>
<th>Parts</th>
<th>Memberships $\mu_1$ and $\mu_2$ for two clusters</th>
<th>$\mu_1$</th>
<th>$\mu_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Screw shaft</td>
<td></td>
<td>0.9883</td>
<td>0.0117</td>
</tr>
<tr>
<td>Steel bar</td>
<td></td>
<td>0.0276</td>
<td>0.9724</td>
</tr>
<tr>
<td>Square steel</td>
<td></td>
<td>0.1236</td>
<td>0.8764</td>
</tr>
<tr>
<td>Counter shaft</td>
<td></td>
<td>0.9902</td>
<td>0.0098</td>
</tr>
<tr>
<td>Hoist plate</td>
<td></td>
<td>0.0045</td>
<td>0.9955</td>
</tr>
</tbody>
</table>

### Table 9

<table>
<thead>
<tr>
<th>Machines</th>
<th>Parts</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZL207</td>
<td>50SiMn</td>
</tr>
<tr>
<td>1</td>
<td>603–637</td>
</tr>
<tr>
<td>2</td>
<td>N</td>
</tr>
<tr>
<td>3</td>
<td>N</td>
</tr>
<tr>
<td>4</td>
<td>S</td>
</tr>
<tr>
<td>5</td>
<td>N</td>
</tr>
<tr>
<td>6</td>
<td>N</td>
</tr>
<tr>
<td>7</td>
<td>N</td>
</tr>
<tr>
<td>8</td>
<td>N</td>
</tr>
</tbody>
</table>

N: non-responsive; S: sand-casting; J: alloy-casting; T: tempering; OC: oil cooled.
binary data. We set the value 1 if the part is processed by the machine and 0 otherwise. Say, for example, that machine 8 had made the “OC” process on the part 50SiMn so that we set the value to 1. The transferred binary machine data are shown in Table 12. Because the binary data are special types of mixed-variable data, MVFCM is still used for these binary data. Thus, we also implement MVFCM for the data of Table 12 and then find the validity indexes for $c = 2, 3, 4,$ and $5$. The results are shown in Table 13. We find that PC, PE, and XB give the optimal cluster number $c^* = 4$. We then implement MVFCM for these eight machine data of Table 12 with $c = 4$. The final MVFCM memberships with $c = 4$ are shown in Table 14 with four machine groups of $\{5, 7, 8\}$, $\{2, 6\}$, $\{1, 4\}$ and $\{3\}$. After we have four part families of $\{50\text{SiMn}, 35\text{SiMn}, 42\text{SiMn}\}$, $\{2\text{L402}, 2\text{L202}\}$, $\{2\text{L207}\}$, and $\{2\text{L205A}\}$ and four machine groups of $\{5, 7, 8\}$, $\{2, 6\}$, $\{1, 4\}$ and $\{3\}$, we can construct the part–machine cells. These four cells are as follows:

Cell 1: $\{50\text{SiMn}, 35\text{SiMn}, 42\text{SiMn}\}$ with $\{5, 7, 8\}$;  
Cell 2: $\{2\text{L402}, 2\text{L202}\}$ with $\{2, 6\}$;  
Cell 3: $\{2\text{L207}, 2\text{L205A}\}$ with $\{1, 4\}$;  
Cell 4: $\{50\text{SiMn}, 35\text{SiMn}, 42\text{SiMn}\}$ with $\{3\}$.

The results are shown in Table 15. However, it is reasonable to incorporate cell 1 and cell 4 into one cell. In this case, we have three part–machine cells as follows:

Cell 1: $\{50\text{SiMn}, 35\text{SiMn}, 42\text{SiMn}\}$ with $\{3, 5, 7, 8\}$;  
Cell 2: $\{2\text{L402}, 2\text{L202}\}$ with $\{2, 6\}$;  
Cell 3: $\{2\text{L207}, 2\text{L205A}\}$ with $\{1, 4\}$.

The results are shown in Table 16. These actually present good final results.

### Table 10
Validity indexes for seven parts from Table 9

<table>
<thead>
<tr>
<th>Cluster numbers</th>
<th>Validity indexes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PC(c)</td>
</tr>
<tr>
<td>$c = 2$</td>
<td>0.8524</td>
</tr>
<tr>
<td>$c = 3$</td>
<td>0.8875</td>
</tr>
<tr>
<td>$c = 4$</td>
<td>0.9195</td>
</tr>
<tr>
<td>$c = 5$</td>
<td>1.0400</td>
</tr>
<tr>
<td>$c = 6$</td>
<td>0.9850</td>
</tr>
</tbody>
</table>

### Table 11
MVFCM memberships of four part families for seven parts from Table 9

<table>
<thead>
<tr>
<th>Part family</th>
<th>Parts</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ZL207</td>
</tr>
<tr>
<td>$c1$</td>
<td>0.00004</td>
</tr>
<tr>
<td>$c2$</td>
<td>0.00005</td>
</tr>
<tr>
<td>$c3$</td>
<td>0.99973</td>
</tr>
<tr>
<td>$c4$</td>
<td>0.00018</td>
</tr>
</tbody>
</table>

### Table 12
Transferred binary data for eight machines from Table 9

<table>
<thead>
<tr>
<th>Machines</th>
<th>Parts</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ZL207</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
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<tr>
<td>5</td>
<td>0</td>
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<tr>
<td>6</td>
<td>0</td>
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<tr>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
</tr>
</tbody>
</table>
7. Conclusions

GT is a useful way of increasing the productivity for manufacturing high-quality products and improving the flexibility of manufacturing systems. CF is an important GT procedure. It can form part families, machine groups and part–machine cells. There are many useful CF methods in the literature. However, most are only for numeric data, especially binary data. In fact, part or machine data are often

Table 13
Validity indexes for eight machines from Table 12

<table>
<thead>
<tr>
<th>Cluster numbers</th>
<th>Validity indexes</th>
<th>PC(c)</th>
<th>PE(c)</th>
<th>XB(c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>c = 2</td>
<td></td>
<td>0.9437</td>
<td>0.4811</td>
<td>0.5245</td>
</tr>
<tr>
<td>c = 3</td>
<td></td>
<td>0.9519</td>
<td>0.5002</td>
<td>0.2070</td>
</tr>
<tr>
<td>c = 4</td>
<td></td>
<td>1.0328</td>
<td>0.3119</td>
<td>0.0716</td>
</tr>
<tr>
<td>c = 5</td>
<td></td>
<td>0.9548</td>
<td>0.5816</td>
<td>0.0952</td>
</tr>
</tbody>
</table>

Table 14
MVFCM memberships of four groups for eight machines from Table 12

<table>
<thead>
<tr>
<th>Groups</th>
<th>Machines</th>
</tr>
</thead>
<tbody>
<tr>
<td>c1</td>
<td>0.01571</td>
</tr>
<tr>
<td>c2</td>
<td>0.02060</td>
</tr>
<tr>
<td>c3</td>
<td>0.80357</td>
</tr>
<tr>
<td>c4</td>
<td>0.16012</td>
</tr>
</tbody>
</table>

Table 15
Part–machine cell formation with four cells based on MVFCM

<table>
<thead>
<tr>
<th>Machines</th>
<th>Parts</th>
</tr>
</thead>
<tbody>
<tr>
<td>50SiMn</td>
<td>35SiMn</td>
</tr>
<tr>
<td>42SiMn</td>
<td>ZL402</td>
</tr>
<tr>
<td>ZL202</td>
<td>ZL207</td>
</tr>
<tr>
<td>ZL205A</td>
<td></td>
</tr>
<tr>
<td>6 N N N</td>
<td>N</td>
</tr>
<tr>
<td>7 N N N</td>
<td>N</td>
</tr>
<tr>
<td>8 N N N</td>
<td>N</td>
</tr>
</tbody>
</table>

Table 16
Part–machine cell formation with three cells based on MVFCM

<table>
<thead>
<tr>
<th>Machines</th>
<th>Parts</th>
</tr>
</thead>
<tbody>
<tr>
<td>50SiMn</td>
<td>35SiMn</td>
</tr>
<tr>
<td>42SiMn</td>
<td>ZL402</td>
</tr>
<tr>
<td>ZL202</td>
<td>ZL207</td>
</tr>
<tr>
<td>ZL205A</td>
<td></td>
</tr>
<tr>
<td>6 N N N</td>
<td>N</td>
</tr>
<tr>
<td>7 N N N</td>
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<td>8 N N N</td>
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presented, except in numeric, with other types such as symbolic and fuzzy. Recently, Yang et al. (2004) proposed a fuzzy clustering algorithm for these mixed types of data, called mixed-variable fuzzy c-means (MVFCM). Our objective in this paper is to apply the MVFCM algorithm to CF when the part–machine matrix has mixed-variable data types. Because real data in cellular manufacturing systems vary, numeric data may not be enough for the part–machine matrix in real cases. Symbolic and fuzzy data may also be represented. In these cases, the MVFCM algorithm is very useful. These data have been described and constructed in this paper. We also give several efficient real examples where we use MVFCM in CF with mixed-variable data types.

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References


